CONTINUITY DIFFERENTIABILITY

1. DEFINITION

If the graph of a function has no break or jump, then it is said to be continuous function. A function which is not continuous is called a discontinuous function.

2. CONTINUITY OF A FUNCTION AT A POINT

A Function f(x) is said to be continuous at some point x=a of its domain if

$$\lim_{x \to a} f(x) = f(a)$$

i.e., If
$$\lim_{x\to a-0} f(x) = \lim_{x\to a+0} f(x) = f(a)$$

i.e., If
$$f(a-0) = f(a+0) = f(a)$$

i.e., If $\{LHL \text{ at } x=a\} = \{RHL \text{ at } x=a\} = \{\text{ value of the function at } x=a\}$.

3. CONTINUITY FROM LEFT AND RIGHT

Function f(x) is said to be

- (i) Left Continuous at x=a if $\lim_{x\to a-0} f(x) = f(a)$ i.e. f(a-0) = f(a)
- (ii) Right Continuous at x=a if $\lim_{x\to a+0} f(x) = f(a)$ i.e. f(a+0) = f(a)

Thus a function f(x) is continuous at a point x=a if it is left continuous as well as right continuous at x=a.

4. CONTINUITY IN AN INTERVAL

- (1) A function f(x) is continuous in an open interval (a, b) if it is continuous at every point of the interval.
- (2) A function f(x) is continuous in a closed interval [a, b] if it is
 - (i) continuous in (a, b)
 - (ii) right continuous at x=a
 - (iii) left continuous at x=b

5. CONTINUOUS FUNCTIONS

A function is said to be continuous function if it is continuous at every point in its domain. Following are examples of some continuous functions:

(i)
$$f(x) = x$$
 (Identify function)

(ii)
$$f(x) = c$$
 (Constant function)

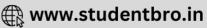
(iii)
$$f(x) = a_0 x^n + a_1 x^{n-1} + \dots + a^n$$
 (Polynomial function)

(iv)
$$f(x) = \sin x, \cos x$$
 (Trigonometric function)

(v)
$$f(x) = a^x$$
, e^x , e^{-x} (Exponntial function)

(vi)
$$f(x) = log x$$
 (Logarithmic function)





(vii)
$$f(x) = \sinh x, \cosh x, \tanh x$$

(Hyperbolic function)

$$(viii) f(x) = |x|, x + |x|, x - |x|, x |x|$$

(Absolute value functions)

6. DISCONTINUOUS FUNCTIONS

A function is said to be a discontinuous function if it is discontinuous at atleast one point in its domain. Following are examples of some discontinuous functions:

No.

Functions

Points of discontinuity

(i)

[x]

Every Integers

(ii)

x-[x]

Every Integers

(iii)

 $\frac{1}{x}$

x = 0

(iv)

tanx, secx

 $X = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots$

(v)

cotx, cosecx

x = 0, $\pm \pi$, $\pm 2\pi$,.....

(vi)

 $\sin\frac{1}{x}$, $\cos\frac{1}{x}$

x = 0

(vii)

 $e^{1/x}$

x = 0

(viii)

coth x , cosechx

x = 0

7. PROPERTIES OF CONTINUOUS FUNCTIONS

The sum, difference, product, quotient (if $Dr \neq 0$) and composite of two continuous functions are always continuous functions. Thus if f(x) and g(x) are continuous functions then following are also continuous functions:

(i)
$$f(x)+g(x)$$

(ii)
$$f(x)-g(x)$$

(iii)
$$f(x).g(x)$$

(iv) $\lambda f(x)$, where λ is a constant

(v)
$$\frac{f(x)}{g(x)}$$
, if $g(x) \neq 0$

(vi) f[g(x)]

8. IMPORTANT POINT

The discontinuity of a function f(x) at x = a can arise in two ways

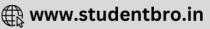
- (i) If $\lim_{x\to a^-} f(x)$ exist but $\neq f(a)$ or $\lim_{x\to a^+} f(x)$ exist but $\neq f(a)$, then the function f(x) is said to have a removable discontinuty.
- (ii) The function f(x) is said to have an unremovable discontinuity when $\lim_{x\to a} f(x)$ does not exist.

i.e.
$$\lim_{x\to a^-} f(x) \neq \lim_{x\to a^+} f(x)$$

Differentiability at a point

Let f(x) be a ral valued function defined on an open interval (a, b) and let $c \in (a, b)$. Then f(x) is said to be





differentiable or derivable at x = c, iff $\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$ exists finitely.

This limit is called the derivative or differential coefficient of the function f(x) at x = c, and is denoted by f'(c) or

Df(c) or
$$\left\{\frac{d}{dx}f(x)\right\}_{x=c}$$

$$\lim_{x \to c^{-}} \frac{f(x) - f(c)}{-h} = \lim_{h \to 0} \frac{f(c+h) - f(c)}{-h}$$

is called the left hand derivative of f(x) at x = c and is denoted by $f'(c^-)$ or Lf'(c) while.

$$\lim_{x \to c^+} \frac{f(x) - f(c)}{x - c} \text{ or } \lim_{h \to 0} \frac{f(c + h) - f(c)}{h}$$

is called the right hand derivative off (x) at x = c and is denoted by $f'(c^+)$ or Rf'(c).

Thus, f(x) is differentiable at $x = c \iff Lf'(c) = Rf'(c)$.

If $Lf'(c) \neq Rf'(c)$ we say that f(x) is not differentiable at x = c.

Differentiability in a set

A fuction f(x) defined on an open interval (a, b) is said tobe differentiable or derivable inopen interval (a, b) if it is differentiable at each point of (a, b)



